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ADDENDUM

Ising spinodal decomposition at $T = 0$ in one to five dimensions

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Abstract. Monte Carlo simulations determine the fraction of not yet flipped spins as a function of time, if the initial spin configuration is random in a nearest-neighbour Ising model. The exponent of Derrida, Bray and Godreche in one and two dimensions is reconfirmed for much larger systems and generalized to three dimensions. In five and more dimensions, this nearest-neighbour Ising model suggests an asymptotically finite fraction of never flipping spins.

Checking whether spins flip in a finite-temperature Monte Carlo simulation is an old method for understanding phase transitions, in particular for spin glasses [1–4]. Derrida, Bray and Godreche recently applied it to zero temperature and found non-trivial power laws for the time dependence of the fraction r_t of spins which have not yet been flipped when starting from a random distribution with half the spins up and half the spins down in an Ising ferromagnet [5]. Even in one dimension, they found $r_t \propto t^{-0.37}$ for long times t. The present addendum tries to answer the question of how this exponent depends on the dimensionality d, and whether the exponents of Derrida *et al* [5] are influenced by finite-size effects.

We work with the usual nearest-neighbour spin-$\frac{1}{2}$ Ising model of L^d spins (ignoring the Potts generalizations of [5]) with Glauber kinetics; at zero temperature that means orienting each spin in the direction of the majority of its neighbours, and to orient it randomly if it has as many up as down neighbours. We work with regular updating instead of the random updating of [5]. While at finite temperatures this choice is unimportant for the equilibrium properties and the dynamical critical exponent [6], in the present $T = 0$ case a different behaviour might be possible. For $2 \leqslant d \leqslant 7$ we use a multi-spin coding technique with one bit per spin [7] where consecutively updated spins are not nearest neighbours. For $d = 1$ we instead used one word per spin and updated the chain consecutively from left to right. Thus possible deviations from [5] should be strongest for the one-dimensional case. If in the case of an equal number of up and down neighbours we orient the spin not randomly but always in one direction, we arrive at a bootstrap percolation model [8] discussed both numerically and analytically [9] and exhibiting logarithmic size effects. Therefore we try much larger system sizes than [5]: $L = 10^9$, 100 800, 1960, 280, and 72 for $d = 1$ to 5 (short simulations were made for 88^5, 40^6, 24^7).

Figure 1 summarizes our simulations. The larger the dimensionality d, the slower the decay with time t of the fraction r_t of spins which have not yet flipped. For long times, r_t decays as $t^{-0.376}$, $t^{-0.220}$, $t^{-0.166}$, respectively, in one to three dimensions, confirming the

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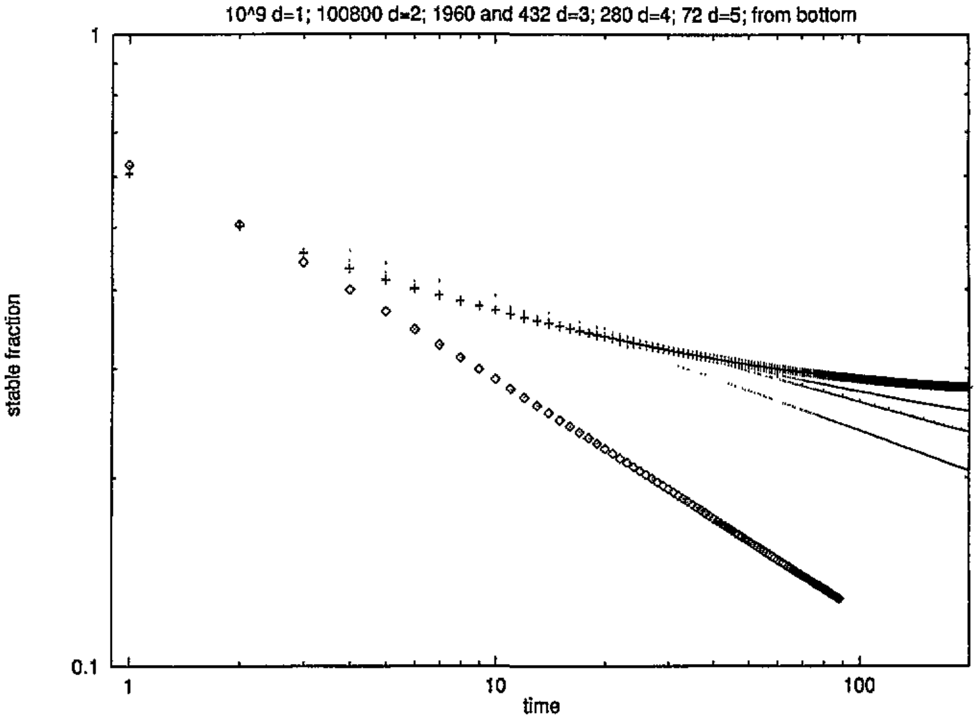

Figure 1. Variation with time of the fraction of spins which have not yet been flipped, in one ($\diamond$, $L = 10^9$) to five (+, $L = 72$) dimensions. The small dots refer, from top to bottom, to four ($L = 280$), three ($L = 432$ and 1960), and two ($L = 100\,800$) dimensions.

exponents -0.37 and -0.22 of [5]. Figure 2 shows more precisely the one-dimensional effective exponent $\mathrm{d}(\log r_t)/\mathrm{d}(\log t)$ (found by least-square fits in consecutive intervals) as a function of time, and suggests an error bar of the order of 10^{-3}. In two dimensions, we found important finite-size effects and sample-to-sample fluctuations for $t > 10^4$ for 300^2 spins, the size used in [5]. If, as discussed in [5], domains grow and then become as large as the system, such behaviour has to be expected. Since Derrida *et al* simulated up to $t = 10^4$ and used only $t < 10^3$ for their exponent determination, their results are not significantly influenced by finite-size effects and agree with ours.

In four dimensions, r_t seems to remain finite or to decay to zero only logarithmically with time. For $d > 4$ we no longer see a clear decay towards zero. For example, at $t = 200$ in $d = 5$, we found $r_t = 0.292, 0.280, 0.277, 0.276, 0.287, 0.278$, and 0.279 for $L = 16$, 24, 32, 40, 48, 56 amd 72 and thus no clear finite-size effect except for $L = 16$. Derrida *et al* suggest a $\sqrt{t}$ law for the domain growth, and in the above two-dimensional example, and similarly in three dimensions, we found finite-size effects at $t \simeq L^2/10$. If that rule also holds in higher dimensions, size effects should not be visible for 72^5 spins at the observation times of figure 1, and cannot explain the deviations from a power-law decay seen there for five dimensions. Thus our data suggest that in five and more dimensions a finite fraction of spins never flip, whereas four dimensions are unclear. Nevertheless, the logarithmic size effects from bootstrap percolation [8] serve as a warning that here, as in other simulations, new effects could become visible for larger systems.

Possibly this numerical dependence on d is merely due to the different number of

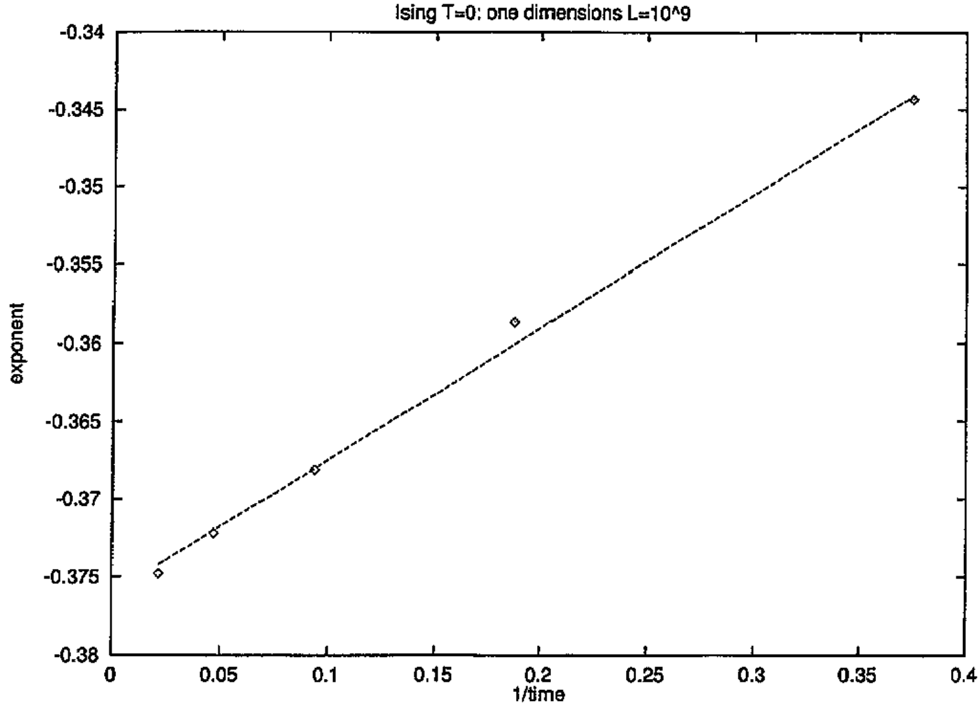

Figure 2. Effective exponent of the time decay in one dimension, versus reciprocal time. Longer simulations in smaller chains are compatible with this result.

neighbours: with increasing d it becomes less and less probable that a spin initially has as many up as down neighbours. For the square lattice with nearest and next-nearest neighbours we recovered the exponent -0.22 for $t > 10^3$, $L = 1001$, whereas for the triangular lattice with six neighbours all spins were flipped after a few hundred steps. (At finite temperatures, eventually all spins flip, and r_t decays exponentially, as we confirmed at $T = T_c$.)

Thus at present $d = 4$ seems to be the border case between low dimensions where the fraction of not yet flipped spins decays with a power of time, and high dimensions where a finite fraction never flips.

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References

[1] Kinzel W 1982 *Z. Physik* B **46** 59
Kohring G A and Schreckenberg M 1991 *J. Physique I* **1** 1087
[2] Liang S 1992 *Phys. Rev. Lett.* **69** 2145
[3] Ray T S and Jan N 1993 *J. Physique I* **3** 2125
Stevens M, Cleary M and D Stauffer 1994 *Physica* **208A**
[4] Gropengiesser U 1994 *J. Physique I* at press
[5] Derrida B, Bray A J and Godreche C 1994 *Preprint*
[6] Ito N 1993 *Physica* **192A** 604

[7] Stauffer D 1993 *Computer Simulation Studies in Condensed Matter Physics VI* ed D P Landau, K K Mon and H B Schuttler (Heidelberg: Springer)
[8] Adler J 1991 *Physica* **171A** 453
Particularly for dynamics see Duarte J A M S 1991 *Physica* **168A** 908
[9] Schonmann R H 1990 *Physica* **167A** 619